LS-42
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Sept. 25, 1985

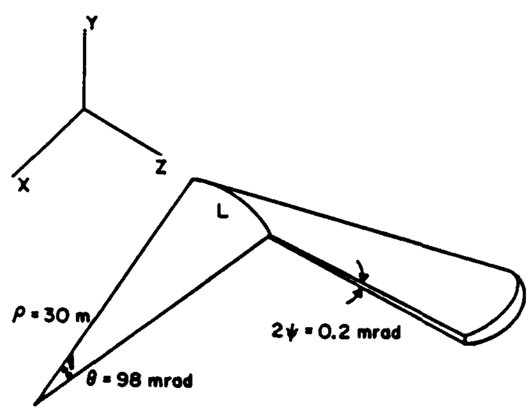
ENERGY AND ANGULAR DISTRIBUTIONS OF RADIATION POWER FROM BENDING MAGNET AND WIGGLER SOURCES AT A 6-GeV RING

Summary:

In order to design radiation ports and beam line components, it is essential to understand the distribution of power from a radiation source as a function of both the photon energy and the solid angle of emission. In this preliminary note, we assemble all the formula involved for the case of a bending magnet and a wiggler. Typical distributions are presented for the case of 6-Gev radiation.

I. Bending Magnet Radiation:

The radiation from a bending magnet source has a uniform distribution in the horizontal (XZ) plane. In the vertical(Y) direction, the radiation is a function of angle ψ, as shown below:



The power radiated per mrad in the XZ plane (or 1 mrad θ) is given by

$$P(\psi)=1.44 \times 10^{-18}[\gamma^5/\rho(m)]\ I(mA)\ F(\gamma\psi)\ \text{watts/mrad}\ \theta/\text{mrad}\ \psi \qquad (1)$$

where

$$\gamma = 1957\ E(GeV) \qquad (2)$$

$$F(\gamma\psi) = (1 + q^2\psi^2)^{-5/2} \quad [7 + 5(\gamma^2\psi^2/(1 + \gamma^2\psi^2))]/16 \qquad (3)$$

$$\rho(m) = 3.335\ E(GeV)/B(T) \qquad (4)$$

For the 6-GeV ring, $\gamma = 11742$, I = 100mA, B = 0.67T, and ρ = 29.9m and in Fig. (1) we present the distribution $P(\psi)$. The distribution has a fullwidth (at half the maximum intensity) of $\gamma\psi = 1.2$, which corresponds to $\psi \approx 0.1$ mrad. The power at $\psi = 0$ is given by

$$P(\psi = 0) = 0.63 \times 10^{-18}\ I(mA)\gamma^5/\rho(m) \qquad (5)$$

in units watts/mrad θ/mrad ψ. In Fig (1), $P(\psi=0)$ = 472 m/mrad θ/mrad ψ.

In Fig.(1), we also show a rectangle of width $\psi = 1.44/\ \gamma = 1.2$ mrad which can be used to represent the total power integrated over all ψ (per mrad θ in the horizontal plane). Proper normalization yields

$$\int_{-\infty}^{\infty} F(\gamma\psi)d\psi = 0.668/\gamma \qquad (6)$$

Integrating Eq.(1) over ψ we obtain the total power per mrad of θ

$$P' \text{ (watts/mrad } \theta) = 9.61\text{x}10^{16}\gamma^{4} I(\text{mA})/\rho(\text{m}) \quad (7)$$

which can be expressed in terms of the ring energy by

$$P' \text{ (watts/mrad } \theta) = 4.33 \text{ X } 10^{-3} \text{ E}^{3} \text{ (GeV}^{3})\text{I(mA)B(T)} \quad (8)$$

If the trajectory of the positron through the bending magnet is of length L(m), the total horizontal angle over which the radiation will emerge is given by $\theta = L/\rho$. Thus the total power from the bending magnet radiation is

$$P \text{ (watts/mrad } \theta) = 1.27 \text{ E}^{2} \text{ (GeV}^{2})\langle B^{2} \text{ (T}^{2})\rangle L(\text{m})I(\text{mA}). \quad (9)$$

For the 6-GeV ring, $P' = 60$ watts/mrad θ, and $P = 6.02$ kWatts/bending magnet since the radiation from the bending magnet source is spread over 98.17 mrad. It should be pointed out that there are 64 such bending magnets which complete the synchrotron trajectory, 98.17 mrad X 64 = 2π.

For above expressions are strictly true for very small positron beam. For sources with appreciable angular spread, σ_y' the power will be reduced approximately by the factor $[1 + (2\gamma^2 \sigma_y'^{\,2})]^{1/2}$. For the 6-GeV source,

$$\sigma_y' = \sqrt{\varepsilon_y/\beta_y}$$

and is $\sigma_y' = 0.031$ mrad. This reduces the power by a factor of 1.24.

Figure 1 also contains the power distribution for the NSLS bending magnet radiation. A significant point to be emphasized (Table 1) is the comparison of the average power and peak power for the two sources. For the beam line design the peak power is more critical than the average power.

TABLE 1.

Average and Peak Power of the BM radiation from 6-GeV and NSLS

	NSLS 2.5 GeV	6-GeV
current (mA)	500	100
2ψ (mrad)	0.41	0.2
Peak power (watts/mrad θ)	128	471
Average power (watts/mrad θ)	40	60
Critical energy (keV)	5.0	16.0

II. Wiggler Radiation:

The critical energy of the photons and the flux can be increased by introducing a wiggler in the straight section. The radiation from a wiggler source has distributions both in ψ and θ.

The power from a wiggler at $\theta = 0$, for all ψ is calculated identically to bending magnet radiation. In fact, it is basically given by Eq. (8), except the power will increase by 2N, in a wiggler with 2N poles (or N wiggler periods). Thus for all ψ, the power radiated in the forward direction is

$$P\ (\theta = 0)\ 8.66 \times 10^{-3}\ E^3\ (\mathrm{GeV}^3) B(T) I(mA) N \text{ watts/mrad } \theta. \qquad (10)$$

The power is given by Eq. (9), except that $\langle B^2 \rangle$ is replaced by $B_o/2$, where B_o is the peak wiggler field.

Hence

$$P\ (\text{watts}) = 0.633\ E^2(\text{GeV}^2)\ B_o^2\ (T^2)\ I(\text{mA})\ L(\text{m}) \qquad (11)$$

where L is the effective wiggler length. We can replace B_o in Eq. (11) by the wiggler deflection parameter K,

$$K = 0.934\ B\ (T)\ \lambda_o\ (\text{cm}) \qquad (12)$$

Since $L(\text{m}) = \lambda_o\ (\text{cm})\ N \times 10^{-2}$, we get

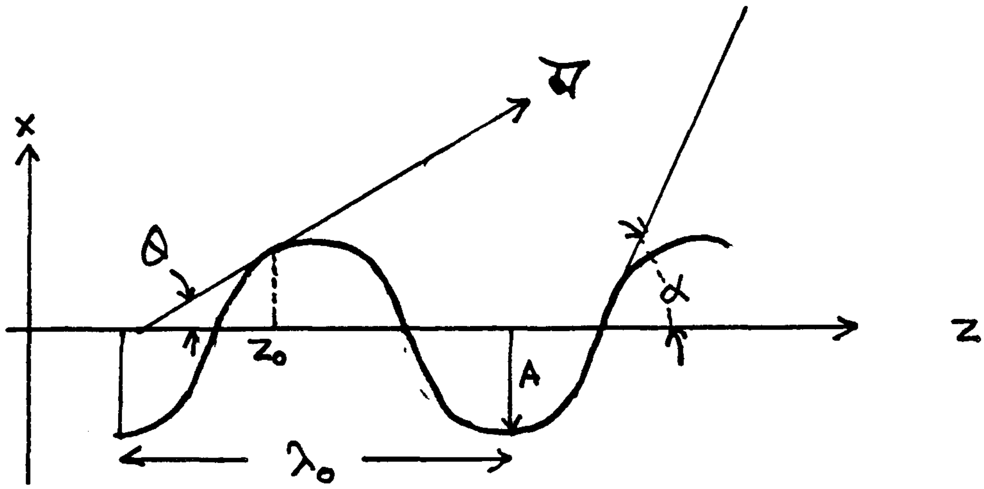
$$P\ (\text{watts}) = 7.25 \times 10^{-3}\ E^2(\text{GeV}^2)\ K^2 N\ I\ (\text{mA})/\lambda_o\ (\text{cm}) \qquad (13)$$

This expression is identical to that for an undulator power.

The horizontal profile of the power (constant for a bending magnet source), will vary for a wiggler. In a sinusoidal wiggler, the field variation along the wiggler axis (z) is

$$B(z) = B_o \cos \alpha \qquad (14)$$

where



$\alpha = 2\pi\ z/\lambda_o$.

The displacement of the positrons in the x-direction is shown above and is given by $x = A \cos \alpha$, and the instantaneous deflection angle $\theta = -\delta \sin \alpha$, where $\delta = 2\pi A/\lambda_o = K/\gamma$.

The value of viewing angle of radiation at $Z=Z_o$ on the trajectory is given by $\theta = -(K/\gamma)\sin \alpha$ and $\theta_{max} = -(K/\gamma)$. The variation in the wiggler field can be expressed by

$$B = B_o \cos \alpha = B_o \cos [\sin^{-1} (\gamma\theta/K)]. \tag{16}$$

The angular distribution of the radiation in the horizontal and vertical planes is now given by

$$P\ (\psi,\theta) = 0.0248\ E^4\ (GeV^4)B(T)NI(mA)\ G(\psi,\theta) \tag{17}$$

$$G(\psi,\theta) = F(\gamma\psi)\ \cos[\sin^{-1}(\gamma\theta/K)] \tag{18}$$

$P(\psi,\theta)$ is in the units watts/mrad θ/mrad ψ. Integrating for all ψ, the distribution of wiggler power as a function of θ can be obtained. This is given by

$$P(\theta) = P(\theta = 0) \cos\left[\sin^{-1}(\gamma\theta/K)\right] \qquad (19)$$

For illustration, we use the following parameters for a wiggler:

$E = 6$ GeV

$B = 1.5$ T

$N = 15$ period

$\lambda_o = 10$ cm

$K = 14$

$I = 100$ mA

The total power from this wiggler is 7.67 kW while the power $P(\theta = 0, \psi = 0)$ is 31.6 kW. The critical photon energy at normal viewing for this wiggler is 35.9 keV and the flux at the critical energy is $9\text{x}10^{14}$. The maximum value of θ for this wiggler is $K/\gamma = \pm 1.2$ mrad. θ.

In Figs. 2 and 3 the variation of the power is shown as a function of ψ (for $\theta = 0$) and as a function of θ (for $\psi = 0$). It can be shown that half of the total power is radiated between the observing angle 0 and $0.404K/\gamma$. Figure 4 gives a power distribution for the 6-GeV wiggler considered assuming a non-divergent electron beam. Beam divergence has no major influence on the angular spreads. The sharp cut-offs in power with θ (Fig. 3,4) is smeared when beam divergence is convoluted.

It is interesting to compare wigglers on a low and high energy storage ring. If two wigglers, one on a 6-GeV ring and the other on a 3-GeV ring, produce photons with the same critical energy, the magnets on the 3-GeV ring should provide larger fields.

$$B_o\ (3\ \mathrm{GeV}) = 4\ B_o(6\ \mathrm{GeV}).$$

For the same current stored in both the rings and for equal wiggler lengths, the ratio of the average power will be

$$\frac{P_N\ (3\ \mathrm{GeV})}{P_N\ (6\ \mathrm{GeV})} = 4.$$

This is an important aspect of advantage for the high energy ring. On the other hand, the peak power of the wiggler on the 6-GeV ring is 4 times that of the wiggler on the 3-GeV ring.

III. Linear and Surface Power Density

In designing optics and heat-flow problems it is essential to evaluate the power densities. As we have pointed out, with high energy storage rings, because of small θ and ψ, peak power is more pertinent for the above problems.

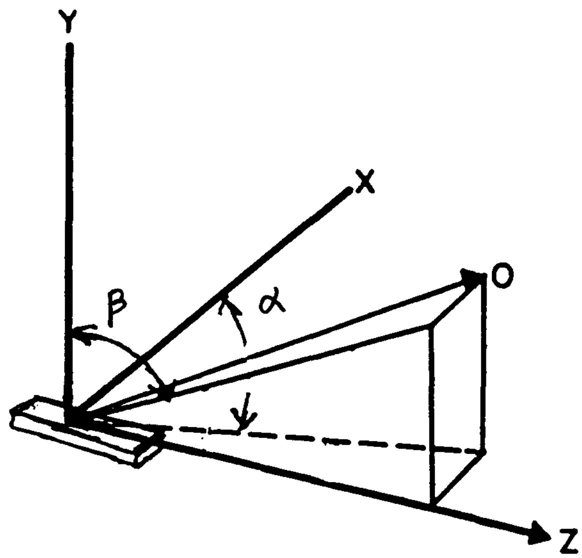
Following Avery [1], the peak power along x-axis (orbital plane) can be integrated over the entire vertical angle (ψ) of radiation to obtain peak power per mrad (θ):

$$P_x\ (\mathrm{w/mrad}) = 8.66\mathrm{x}10^{-3}\ B_o(\mathrm{T})E^3(\mathrm{GeV}^3)I(\mathrm{mA})N \qquad (19)$$

where I is in mA and N is number of periods. The peak power per unit solid angle is

$$P_{xy}\ (\mathrm{w/mrad^2}) = 1.076\mathrm{x}10^{-2}\ B_o(T)E^4(\mathrm{GeV}^4)I(\mathrm{mA})N. \tag{20}$$

The radiation impinging at a distance ρ meters from the source point can be defined in terms of linear and surface power densities, as follows:



Peak Linear Power Density:

$$W_x(\mathrm{w/mm}) = P_x\ \sin\alpha/\ell(\mathrm{m}) \tag{21}$$

Peak Surface Power Density:

$$W_{xy}(\mathrm{w/mm^2}) = P_{xy}\sin\beta/\ell^2(\mathrm{m^2}) \tag{22}$$

The values of these are included in Table 2.

In the above we have not considered the following:

1. The spread in the angular distribution with photon energy.

2. Spatial and angular spreads in the positron beam in the ring.

3. The effect of finite length of the wiggler source.

The influence of the above factors is not significant. The sharp drop in power to zero is at $\theta = \pm 1.2$ mrad will be smeared. At distances large compared to wiggler length ($\ell >> L$, a normal condition at the 6-GeV source), the effect of finite wiggler length is not important.

IV. Variation of Critical Energy E_c of a Wiggler with Normal Angle

One normally defines the critical energy for a wiggler which corresponds to normal viewing angle ($\theta = 0$) which corresponds to $z = \lambda_o/4$ on the sinusoidal trajectory. Buras [2] has shown that

$$\frac{E_c(\theta)}{E_c(0)} = \sin\left(\cos^{-1}\gamma\theta/K\right)$$

We present this for our wiggler in Fig. 5.

Table 2

Comparison of Planned Wigglers on Various Sources

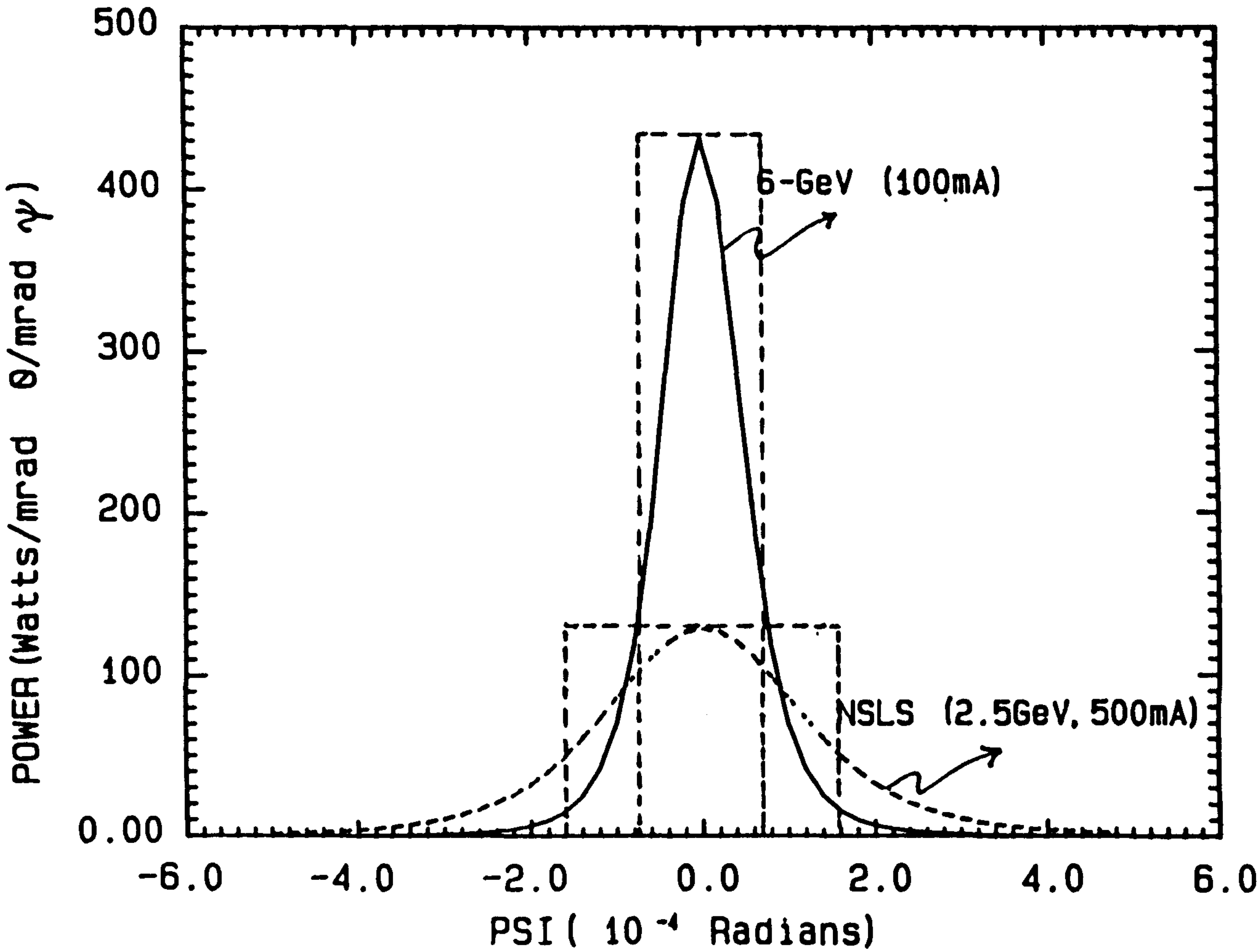
Parameters	SSRL Beamline VIII	NSLS X-17	6-GeV 1.5T
Energy (GeV)	3.0	2.5	6.0
Current (mA)	100	500	100
B_o(T)	1.3	6.0*	1.5
ρ(m)	7.8	1.39	13.3
Poles	30	6	30
λ_o(cm)	12.85	17.4	10.0
E_c(keV)	7.8	24.9	35.9
K	15.6	97.5	14.0
Flux at E_c ($x10^{14}$)	1.4	0.24	2.9
2ψ (mrad)	0.41	0.49	0.2
2θ (mrad)	5.3	39.9	2.4
Average Power (kW)	1.85	37.1	7.7
ℓ(m)	10	10	50
Beam Width 2θ ℓ(mm)	53.0	400.0	120.0
Beam Height 2ψ ℓ(mm)	4.1	4.9	10.0
P_x (w/mrad)	456	1218	4209
P_{xy}(w/mrad^2)	1700	3782	31376
W_x(w/mm) at ℓ	46	122	84
W_{xy}w/mm^2) at ℓ	17	38	13

*Actual design consists of 5 poles with 6T and 2 poles with 2T.

References

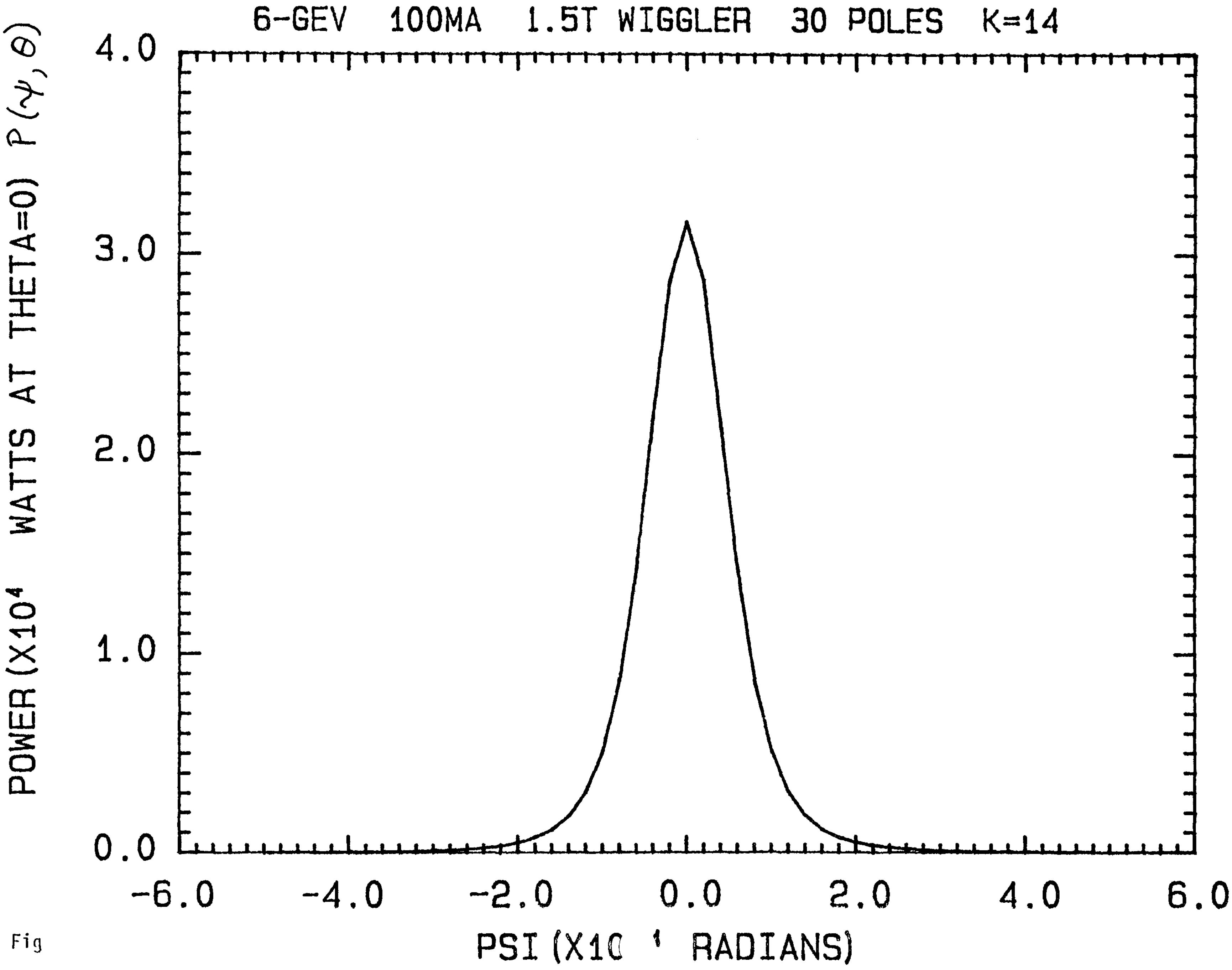
1. R. T. Avery, Nucl. Instrum. Methods 222, 146 (1984).

2. Buras, European Synchrotron Radiation Project Report, 1984.



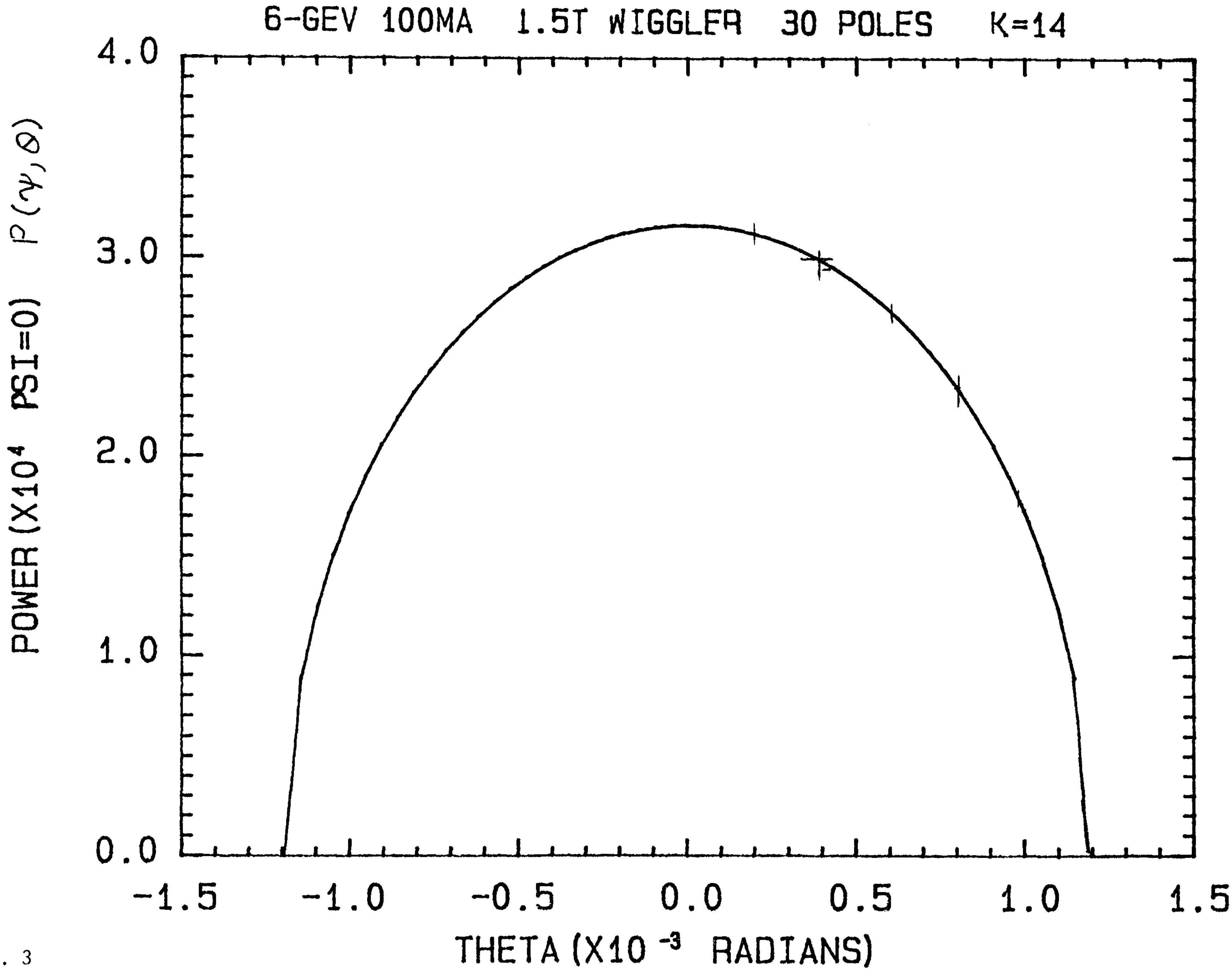
500
400
300
200
100
0.00
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
PSI (10 -4 Radians)
POWER (Watts/mrad θ/mrad ψ)
6-GeV (100mA)
NSLS (2.5GeV, 500mA)

Fig. 1



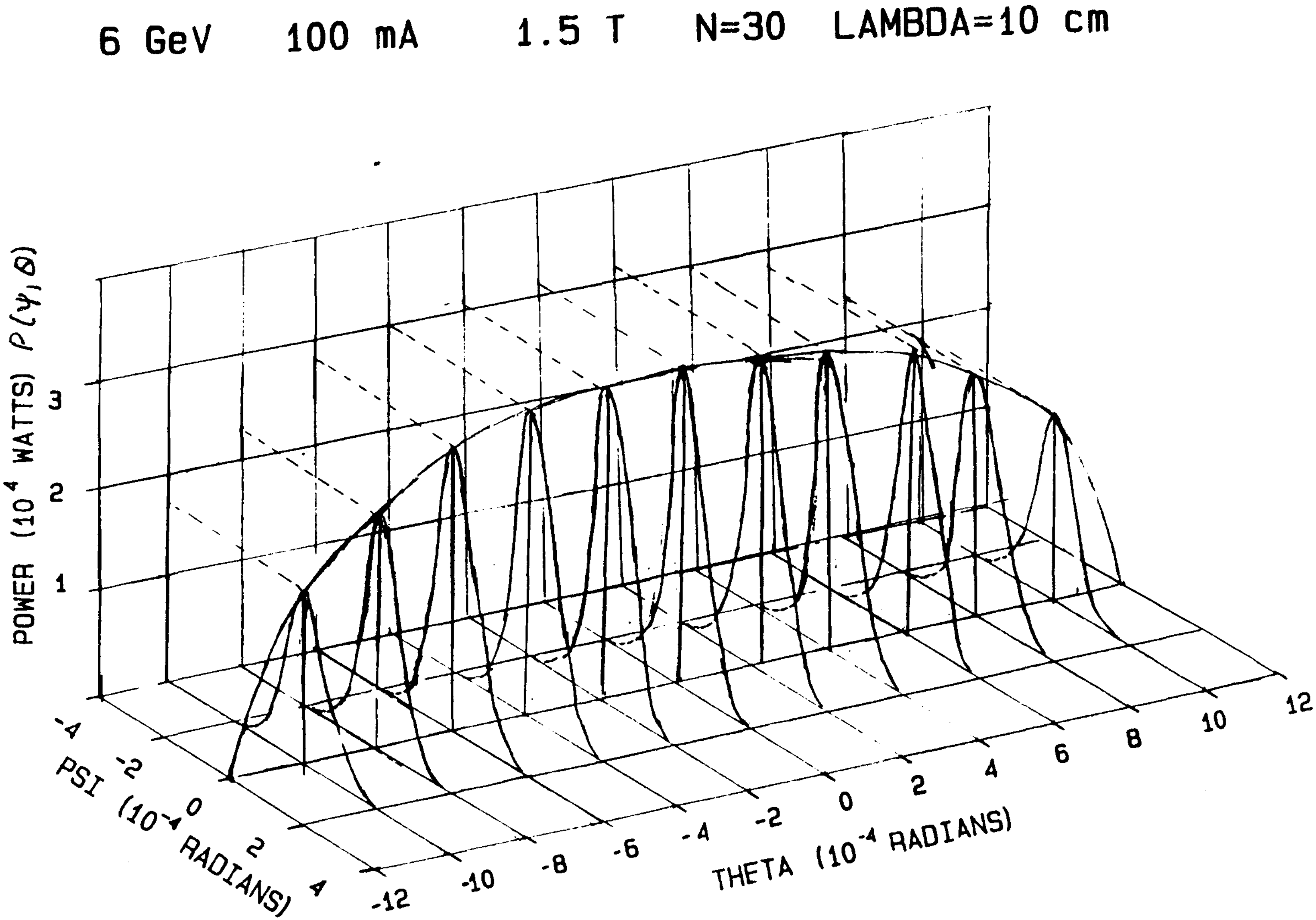
6-GEV 100MA 1.5T WIGGLER 30 POLES K=14
POWER (X10⁴ WATTS AT THETA=0) P(ψ, θ)
4.0
3.0
2.0
1.0
0.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
PSI (X10⁻¹ RADIANS)

Fig



6-GEV 100MA 1.5T WIGGLER 30 POLES K=14
POWER (X10⁴ PSI=0)
P(ψ, θ)
4.0
3.0
2.0
1.0
0.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
THETA (X10⁻³ RADIANS)

Fig. 3



6 GeV 100 mA 1.5 T N=30 LAMBDA=10 cm
POWER (10^4 WATTS) $P(\psi,\theta)$
1
2
3
PSI (10^{-4} RADIANS)
-4
-2
0
2
4
THETA (10^{-4} RADIANS)
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12

Fig. 4

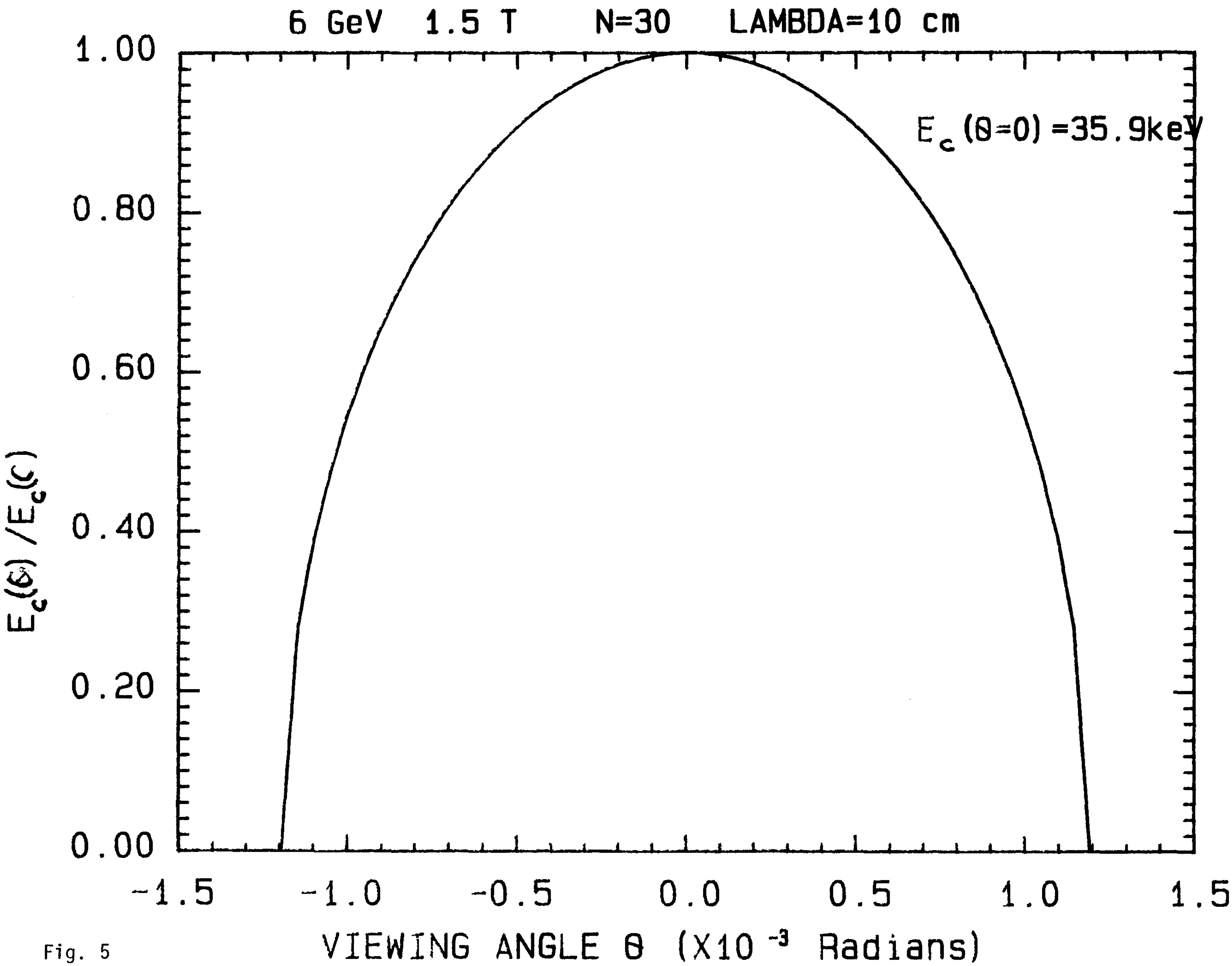

6 GeV 1.5 T N=30 LAMBDA=10 cm
E_c (θ=0) =35.9keV
E_c(θ) /E_c(0)
1.00
0.80
0.60
0.40
0.20
0.00
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
VIEWING ANGLE θ (X10^-3 Radians)

Fig. 5